INDIAN MARITIME UNIVERSITY

(A Central University Government of India)

END SEMESTER EXAMINATIONS-June/July 2019

B.Tech(Marine Engineering)

Semester-II

Mathematics-II (UG11T3202)

Date: 27-06-2019 **Maximum Marks: 100**

Duration: 3 hrs **Pass Marks: 50**

Note: i. Use of approved type Scientific Calculator is permitted.

ii. The symbols have their usual meanings.

<u>Section – A</u> **(10x3 = 30 marks)**

<u>(All Questions are Compulsory)</u>

Q.1 (a) Find a_0 , for $f(x) = \begin{cases} 0 & ,-\pi \le x \le 0 \\ \sin x & ,0 \le x \le \pi \end{cases}$.

(b) Find the Laplace Transform of e^{-3t} sin 5t sin 3t.

(c) Find the Laplace Transform of t^2 sin at.

(d) Find the inverse Laplace Transform of $\frac{s^2-10\,s+13}{(s-7)(s^2-5\,s+6)}$.

(e) Find the orthogonal trajectories of the family of Semi-cubical parabolas: $ay^2 = x^3$.

(f) Solve : $(1-x^2)\frac{dy}{dx} + 2xy = x\sqrt{1-\ x^2}$.

(g) Find the Particular Integral of $\frac{d^2y}{dx^2} - 2\ \frac{dy}{dx} + y = x\,e^x \sin x$.

(h) Three students A, B, C write an entrance examination. Their chances of passing are ½, 1/3, ¼ respectively. Find the probability that at least one of them passes.

(i) Find mean and variance for the following distribution

$f(x) = 1; \quad 0 < x < 1$

(j) Find Mean, Median and Mode of the discrete random variable x whose probability distribution is given below:

$X=x_i$	1	2	3	4	5	6
$P(X=x_i)$	0.25	0.21	0.51	0.01	0.01	0.01

Section – B **(14x5 = 70 marks)**

(Answer any 5 of the following)

Q.2 (a) Find the Fourier series expansion of $f(x) = 2x - x^2$ in (0,3) and hence deduce that $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots = \frac{\pi^2}{12}$. (8 marks)

(b) Show that for $-\pi < x < \pi$,

$$\sin ax = \frac{2\sin a\pi}{\pi}\left(\frac{\sin x}{1^2 - a^2} - \frac{2\sin 2x}{2^2 - a^2} + \frac{3\sin 3x}{3^2 - a^2} - \cdots\right).$$ (6 marks)

Q.3 (a) Evaluate $\int_0^\infty \frac{e^{-at} - e^{-bt}}{t}\,dt$. (6 marks)

(b) Find $L^{-1}\left(\log\frac{3(s^2-2)^2}{2s^5}\right)$. (8 marks)

Q.4 (a) Solve $xy(1 + xy^2)\frac{dy}{dx} = 1$. (6 marks)

(b) Solve by the method of variation of parameters,

$y'' - 2y' + y = e^x \log x.$ (8 marks)

Q.5 (a) A business man goes to hotels X, Y, Z, 20%, 50%, 30% of the time respectively. It is known that 5%, 4%, 8% of the rooms in X, Y, Z hotels have faulty plumbing. What is the probability that business man's room having faulty plumbing is assigned to hotel Z. (7 marks)

(b) Find Moment generating function of the exponential distribution

$f(x) = 1/c \ e^{-x/c}$; $0 \leq x \leq \infty$, $c>0$.

Use this Moment generating function to find mean, variance and standard Deviation. (7 marks)

Q.6 (a) From a box containing 100 transistors 20 of which are defective, 10 are selected at random. Find the probability that, (i) All will be defective (ii) All will be non –defective, (iii) At least one is defective. (7 marks)

(b) In a certain factory producing cycle tyres there is a small chance 1 in 500 for any tyre to be defective. The tyres are supplied in the lots of 20. Using Poisson's distribution calculate the approximate number of lots containing no defective, one defective and two defective tires respectively in a consignment of 20,000 tires. (7 Marks)

Q.7 (a) Obtain the Fourier expansion of x sin x as a cosine series in $(0,\pi)$.

(6 marks)

(b) Solve : $x^3\frac{d^3y}{dx^3}+2x^2\frac{d^2y}{dx^2}+2y=10(x+\frac{1}{x})$ (8 marks)

Q.8 (a) Solve the following equation by the transform method:

$y'' + 4y' + 3y = e^{-t}$, $y(0) = y'(0) = 1$ (7 marks)

(b) Evaluate $L^{-1}\left(\frac{s}{(s^2+1)(s^2+4)(s^2+9)}\right)$. (7 marks)